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A kinetic ANNNI model

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Abstract. We present a kinetic lattice system that evolves in general towards steady non-equilibrium states due to dynamical conflict between nearest- and next-nearest-neighbour interactions. Under two simple particular limits, the system would reach asymptotically the canonical equilibrium states for the ordinary Ising model and for the axial next-nearest-neighbour Ising (ANNNI) model, respectively. We find more generally that, independently of the lattice dimension, the steady state probability distribution for a given class of transition rates has a *quasi-cononical* structure with a short-range *effective Hamiltonian*. We solve exactly the one-dimensional version of this case, and compare its behaviour to the one for the ordinary ANNNI model. In particular, the system is shown to exhibit several spatially modulated phases and *impure* critical points. We also obtain some information on the phase diagram for the two-dimensional lattice.

The one-dimensional lattice system called the axial next-nearest-neighbour Ising or ANNNI model (Elliot 1961; for reviews see Liebmann 1986, Selke 1992) consists of a linear chain with Ising spins, i.e. $s_z = \pm 1$ at each lattice site $z = 1, 2, \ldots$. Typically, interactions are ferromagnetic, say $J_{\mathrm{NN}} > 0$, between any pair of nearest-neighbour (NN) spins, (s_z, s_{z+1}), and antiferromagnetic, $J_{\mathrm{NNN}} < 0$, between any pair of next-nearest-neighbour (NNN) spins, (s_z, s_{z+2}). The d-dimensional (simple-hyper-cubic) ANNNI model simply consists of one-dimensional ANNNI chains that connect to each other by NN interactions of strength J_0 in all the $2(d-1)$ traversal directions. This is supposed to describe, for example, the spatially modulated phase in certain magnetic substances such as erbium. It is also of considerable fundamental interest. That is, the ANNNI model contains microscopic disorder and *frustration* whose study defies the standard methods of statistical physics. We define here a novel kinetic version of the ANNNI model, and present some results for the simplest version of it; most of our results are exact.

Consider a d-dimensional simple-*cubic* lattice, Ω, with spin (equivalently, particle, etc.) configurations $s \equiv \{s_r = \pm 1; r \in \Omega\}$. There are interactions of strength J_{NN} between NN, as in the Ising Hamiltonian, $H(s) = -J_{\mathrm{NN}} \sum_{|r-r'|=1} s_r s_{r'}$. The probability of s changes with time according to the Markovian equation

$$\partial P(s;t)/\partial t = \sum_{s^r} [c(s|s^r)P(s^r;t) - c(s^r|s)P(s;t)] \tag{1}$$

where, unlike for the ordinary canonical Ising model, the transition rate describes a conflict between different tendencies at temperature T. More explicitly, if s^r represents s with the sign of the spin variable at r changed (due to a *flip* $s_r \to -s_r$), the probability that s transforms into s^r is a superposition of elementary events, or

$$c(s^r|s)=\int_{-\infty}^{+\infty} \mathrm{d}\, J_{\mathrm{NNN}} f(J_{\mathrm{NNN}})\,\omega(s^r|s;J_{\mathrm{NNN}}). \tag{2}$$

Here, J_{NNN} is a random variable of distribution $f(J_{\mathrm{NNN}})$ normalized to unity. We assume for the elementary transition rate that

$$\omega(s^r|s;J_{\mathrm{NNN}})=\Psi(\beta\Delta H_{J_{\mathrm{NNN}}}) \tag{3}$$

where $\beta=(k_{\mathrm{B}}T)^{-1}$, k_{B} is Boltzmann's constant, $\Delta H_{J_{\mathrm{NNN}}}\equiv H(s^r;\ J_{\mathrm{NN}}, J_{\mathrm{NNN}})-H(s;J_{\mathrm{NN}},J_{\mathrm{NNN}})$, and

$$H(s;J_{\mathrm{NN}};J_{\mathrm{NNN}})=-J_{\mathrm{NN}}\sum_{|r-r'|=1} s_r s_{r'}-J_{\mathrm{NNN}}\sum_{|r-r'_z|=2} s_r s_{r'_z} \tag{4}$$

where the last sum is over the NNN of $r\equiv(x,y,z)$ in the z-direction, denoted r'_z. Except for the fact that $J_0=J_{\mathrm{NN}}$ for simplicity here (see, however, below after (12) for a different case), (4) is *formally* identical to the Hamiltonian that characterizes the ordinary ANNNI model. Nevertheless, only the parameter J_{NN} in (4) is a constant; according to (2), J_{NNN} is a random variable that one may interpret that varies during time evolution. That is, (4) is not a real Hamiltonian for the system but decides kinetics via (3).

The function Ψ in (3) is arbitrary. We are interested here, however, in the choice

$$\Psi(X)=\mathrm{const}\cdot\exp(-\tfrac{1}{2}X). \tag{5}$$

This restriction results in part from the requirement that we want (3) to satisfy the condition of detailed balance, namely $\Psi(X)=\Psi(-X)\exp(-X)$. Then, the system (1)–(5) has two simple equilibrium limits for which detailed balance is satisfied also by the superposition (2). The 'trivial' case occurs for $J_{\mathrm{NNN}}\equiv 0$; then, (4) transforms into the Ising Hamiltonian, and one has that $c(s^r|s)=\omega(s^r|s;0)$ from (2); consequently, the system reduces to the familiar kinetic version of the Ising model (Glauber 1963).For $f(J_{\mathrm{NNN}})=\delta(J_{\mathrm{NNN}}\text{-const})$, where δ represents the Dirac function, (1)–(5) represent the simplest kinetic version of the ANNNI model one may think of, i.e. it leads asymptotically to the equilibrium state for temperature T and energy (4) with $J_{\mathrm{NNN}}=\mathrm{const}$. The situation is less simple otherwise given that (2) implies in general that the interaction to second neighbours, J_{NNN}, does not have a single value but is sampled from $f(J_{\mathrm{NNN}})$ *at each step* during time evolution. The resulting system generalizes the ANNNI model, and is a novel example of a class of stochastic Ising systems with competing interaction kinetics introduced in Garrido and Marro (1989). One may interpret that the (random) variations of J_{NNN} during evolution are due to very fast and completely random diffusion of impurities (of the sort that one may expect to occur in magnetic substances, for instance (cf Garrido and Marro 1994)). Anyhow, such a conflict between (NNN) interactions will lead the system generally to a steady non-equilibrium state whose nature is unknown, e.g. (non-equilibrium) phase transitions and critical phenomena may occur having a complex relation (if any) to the corresponding equilibrium situations. Once the rest of the system parameters (e.g. the structure of and the constants in (4)) are given, the nature of the resulting phenomena will depend, even strongly, on the choices for $f(J_{\mathrm{NNN}})$ and $\Psi(X)$ determining the effective transition rate (2) (i.e. the latter will not satisfy detailed balance in general). It is our aim here to describe a simple case of (1)–(4) for which exact results may be derived for any lattice dimension d; in fact, it was for the sake of simplicity that the exponential structure in (5) was selected.

The following result applies. The steady state for system (1)–(5) has, say, a *quasi-canonical* structure for any dimension d. We mean that the limit of $P(s;t)$ in (1) as $t\to\infty$ is $P_{st}(s)=Z^{-1}\exp[-E(s)]$, where $Z\equiv\Sigma_s\exp[-E(s)]$. Here, $E(s)$ is formally indentical to (4) but involves different effective coupling constants. We may express this by writing $E(s)=H(s;K_1;K_2)$; then, one gets

$$K_1=\beta J_{\mathrm{NN}} \tag{6a}$$

$$\tanh(2K_2)=[\![\sinh(2\beta J_{\mathrm{NNN}})]\!][\![\cosh(2\beta J_{\mathrm{NNN}})]\!]^{-1} \tag{6b}$$

where $[\![\ ]\!]$ denotes the average with $f(J_{\mathrm{NNN}})$ defined by (2). The steady state is unique given that *effective detailed balance* holds for (5), i.e. $c(s'|s)\exp[-E(s)]=c(s|s')\exp[-E(s')]$. The proof is a matter of algebra (the simplest way to verify this is to assume $E(s)=H(s;K_1;K_2)$, and look for expressions (6) by requiring effective detailed balance). This amounts to an extension to interactions other than the NN Ising ones of the method in López-Lacomba *et al* (1990) (to where we refer the reader for further details). This result holds *for any d* due to our choice (5). *Almost* any other $\Psi(X)$ would imply full non-equilibrium behaviour in general; this occurs, for instance, if $\Psi(X)$ in (3) corresponds either to the rate used by Glauber (1963) to set up the kinetic version of the Ising model, or to the Metropolis algorithm which is familiar in Monte Carlo studies.

In principle, one may study the consequences of the result in the previous paragraph for any d. We first refer here to the case $d=1$. In fact, our study may be pursued by exact analytical methods (involving no assumption) for $d=1$ only. Furthermore, the one-dimensional version of this (axial) system is expected to exhibit some of its most relevant features, as occurs for the ordinary ANNNI model. A specific question is the phase diagram for our generalized ANNNI model that results from (6), and its comparison with the one for the ordinary case that follows from (4) if J_{NNN} represents a constant. The latter has been studied by Stephenson (1970), Hornreich *et al* (1979) and Tanaka *et al* (1987), for instance.

The formal similarity between the two problems allows us to write $Z=\mathrm{Tr}[M^N]$ for a chain of N spins with periodic boundary conditions. M is the transfer matrix of eigenvalues

$$\lambda_{1,2}=\exp(K_2)\{\cosh(K_1)\pm[\sinh^2(K_1)+\exp(-4K_2)]^{1/2}\} \tag{7a}$$

$$\lambda_{3,4}=\exp(K_2)\{\sinh(K_1)\pm[\cosh^2(K_1)-\exp(-4K_2)]^{1/2}\}. \tag{7b}$$

The correlation function ensues as

$$G(n)=\langle s_z s_{z+n}\rangle=\tfrac{1}{2}\lambda_1^{-n}[(1+\varepsilon)\lambda_3^n+(1-\varepsilon)\lambda_4^n] \tag{8}$$

where $\varepsilon\equiv\frac{1}{2}\sinh(2K_1)/\{[\sinh^2(K_1)+\exp(-4K_2)][\cosh^2(K_1)+\exp(-4K_2)]\}^{1/2}$. Following Stephenson (1970), one may distinguish two well-defined cases:

(i) For $\cosh(K_1)>\exp(-2K_2)$, the four eigenvalues of M are real and simple. One obtains that $G(n)\sim\frac{1}{2}(1+\varepsilon)(\lambda_3/\lambda_1)^n$ as $n\to\infty$, and the correlation length is $\xi\sim1/\ln(\lambda_1/\lambda_3)$.

(ii) For $\cosh(K_1)<\exp(-2K_2)$, two eignenvalues of M are real and simple and the other two are complex. Then, $G(n)\sim|1+\varepsilon||\lambda_3/\lambda_1|^n\cos(n\varphi+\Theta)$, where $\varphi\equiv\tan^{-1}[\mathrm{Im}(\lambda_3)/\mathrm{Re}(\lambda_3)]$ and $\Theta\equiv\tan^{-1}[\mathrm{Im}(1+\varepsilon)/\mathrm{Re}(1+\varepsilon)]$, and one has that $\xi\sim1/\ln|\lambda_1/\lambda_3|$.

This is known to imply an interesting behaviour for the ordinary ANNNI model which corresponds to $K_1=\beta J_{NN}$ and $K_2=\beta J_{NNN}$. That is, $G(n)$ has then a pure exponential decay for (i), while it exhibits oscillations of wavevector φ whose amplitude decays exponentially for (ii). The *disorder line* or boundary between the two regions occurs for values of β that satisfy $\cosh(\beta J_{NN})=\exp(-2\beta J_{NNN})$; consequently, it is required that $J_{NNN}=-|J_{NNN}|<0$, and the boundary occurs at temperature $T=T^*(\alpha)$ that depends on the value for $\alpha\equiv -J_{NNN}/J_{NN}>0$. If we define the parameter $\tau\equiv\exp[-2\beta|J_{NN}+2J_{NNN}|]$, it follows that $\xi\sim\frac{1}{2}\tau^{-1}$ for $\alpha<\frac{1}{2}$ and $\xi\sim 2\tau^{-1/2}$ for $\alpha>\frac{1}{2}$; one may interpret this as corresponding to critical indexes $\nu=1$ and $\frac{1}{2}$, respectively. For $\alpha=\frac{1}{2}$ and $T=0$ (the so-called *frustration point*), the correlation length remains finite; it reflects the fact that the *pure* critical point washes out due to the competition between NN and NNN interactions.

The above picture essentially remains for (6). The implications of the latter are more varied, however, due to the complex dependence of the parameter K_2 on temperature. (We leave to the concerned reader the study of many one-dimensional cases that we do not consider explicitly here, e.g. the ones for the rate functions $\Psi(X)=\min(1, e^{-X})$ and $\Psi(X)=1/(1+e^X)$ that correspond to the Metropolis and Glauber cases, respectively, for which (6) does not hold.) Besides the essential dependence on $\Psi(X)$, the situation may be shown to depend strongly on $f(J_{NNN})$. To maintain a close relation to the ordinary ANNNI case, we (only) illustrate below the case corresponding to the distribution

$$f(J_{NNN})=p_+\delta(J_{NNN}-J_+)+p_-\delta(J_{NNN}+J_-)+(1-p)\delta(J_{NNN}) \tag{9}$$

where $J_+, J_->0$, and $p=p_++p_-$. (Further cases of $f(J_{NNN})$ may be worked out exactly, however.) Then, one has for $\beta\to\infty$ that $\exp(-4K_2)\sim(p_-/p_+)\exp[2\beta(J_--J_+)]$ that goes to ∞, (p_-/p_+) or 0 depending on whether J_- is larger, equal or smaller than J_+, respectively. For $\beta\to 0$, one obtains $K_2\sim\beta(p_+J_+-p_-J_-)$. That is, our system behaves at high temperature as the ordinary ANNNI model with $J_{NNN}=p_+J_+-p_-J_-$, while the low-temperature behaviour is equivalent to having $J_{NNN}=\frac{1}{2}(J_+-J_-)$ instead.

It is remarkable that the effective NNN interaction $K_2(T)$ may change sign as one varies T for the simple choice (9). Let us define

$$\eta\equiv -\tfrac{1}{2}(J_+-J_-)/J_{NN} \qquad \gamma\equiv J_+/J_-$$

(which turn out to be the relevant parameters besides p_+ and p_-). If follows that a changeover of the effective NNN interaction occurs from ferromagnetic to antiferromagnetic both for low enough T (according to whether η is negative or positive, respectively) and for high enough T (according to whether one has $\gamma>p_-/p_+$ or $\gamma<p_-/p_+$, respectively). This and, more generally, the fact that K_2/β has a complex dependence on T are at the origin of the qualitative differences we report below between the present model and the ordinary one.

The case $\eta>0$ corresponds to the most familiar version of the ANNNI model, i.e. $J_{NN}>0$ and $J_{NNN}<0$. Looking for the closest relation between the ordinary and generalized model, one may consider $p_+=p_-=\frac{1}{2}$ besides $\eta>0$. Then, our model reduces precisely to the ordinary ANNNI system with $J_{NNN}=\frac{1}{2}(J_+-J_-)$ for any temperature, i.e. K_2/β becomes independent of T in this case. Otherwise, the behaviour of the two models may differ importantly, even for the simple choices (5) and (9). This is illustrated

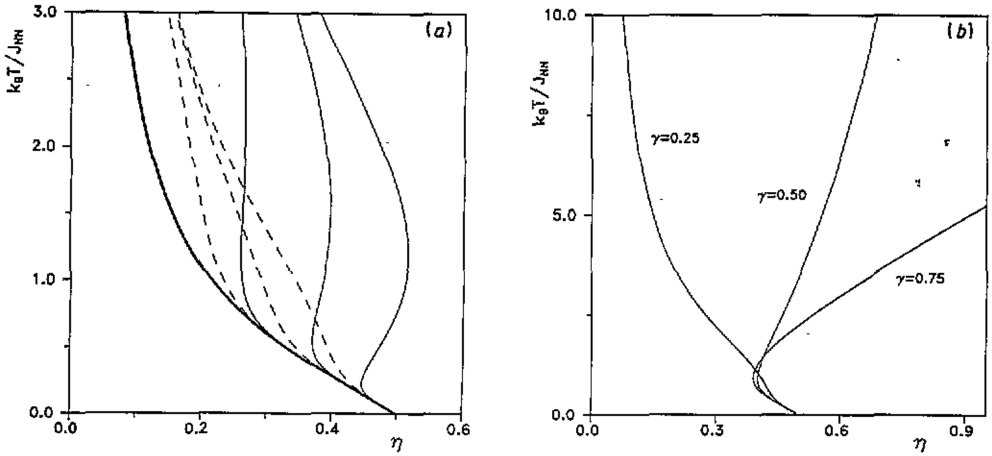
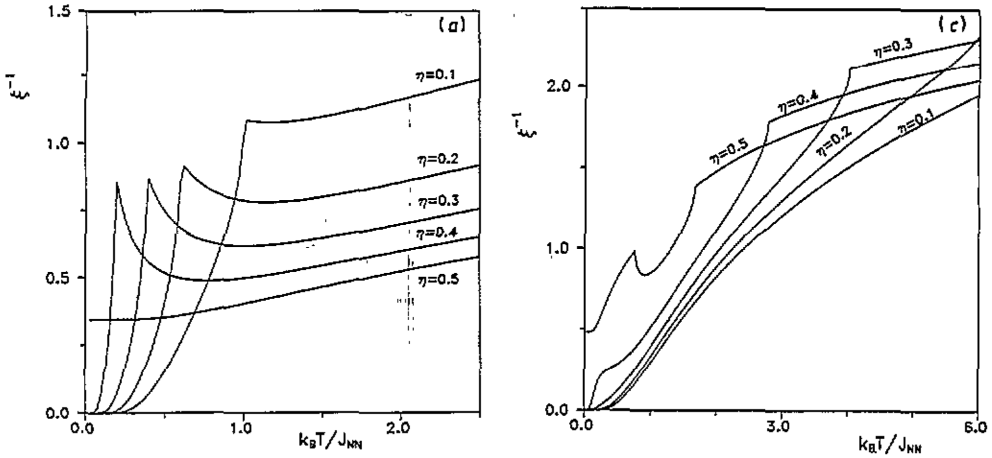


Figure 1. The *disorder line*, $K_1 = K_1(\eta)$, that separates a region to the left in which the correlation function exhibits a pure exponentially decay with distance ($\varphi \equiv 0$) from a region to the right in which spatial correlations decay oscillating with wavevector φ (cf. the main text), as a function of $\eta \equiv -\frac{1}{2}(J_+ - J_-)/J_{NN}$ for different values of $\gamma \equiv J_+/J_-$, p_+ and p_-. This corresponds to exact results for the one-dimensional system. (a) The bold full line is the result for the ordinary ANNNI model that corresponds to the *quasi-canonical* model studied here for $p_+ = p_- = \frac{1}{2}$ and any γ; the broken lines are for $p_+ = p_- = \frac{1}{4}$ and, from left to right, for $\gamma = \frac{3}{4}$, $\frac{1}{2}$ and $\frac{1}{4}$; the full lines are for the same values of γ but for $p_+ = p_- = 0.10$. (b) The case $p_+ = \frac{1}{2}$ and $p_- = \frac{1}{4}$ for indicated values of γ; note that the vertical axis has a different scale from that in part (a).

in figure 1 where the *phase diagram* exhibits important variations with the parameters γ, p_+ and p_-. The situation depicted in figure 1(a), where $\gamma < p_+/p_-$ so that K_2 is antiferromagnetic-like at any temperature, is apparently conventional. It is remarkable, however, that the disorder line for $p_+ = p_- = 0.10$ (but not for $p_+ = p_- = 0.25$) indicates the existence for certain given values of η of two (three if $\gamma = 0.25$) transitions between normal and modulated phases. Even more interesting is the situation in figure 1(b), where different curves correspond to different relations between γ and p_+/p_-; then, K_2 changes in figure 1(b) at $\gamma = \frac{1}{2}$ from antiferromagnetic (for $\gamma < \frac{1}{2}$) to ferromagnetic (for $\gamma > \frac{1}{2}$) at high enough temperature (while it is always antiferromagnetic at low T). One observes in this case a behaviour similar to the one in figure 1(a) for $\eta \approx 0.5$, while the system can only be modulated at low enough T and becomes *normal* for high T if $\eta \gg 0.5$ and γ is large enough. Such a behaviour has no counterpart in the ordinary canonical case. It would be interesting to check if it occurs in nature.

Figure 2 illustrates the correlation length for various cases; cf figure caption. The situation depicted, for instance by figure 2(b) for $\gamma = 0.75$, is remarkable. That is, the line $\eta = 0.3$ reveals that correlations remain monotonic for any T, while for $\eta = 0.4$ a transition occurs to modulated, and then to monotonic behaviour again, as T is increased, and both the second transition and the critical point do not appear for $\eta = 0.5$. On the other hand, it turns out that the most interesting impure picture (and critical behaviour) occurs for $\beta \to \infty$, as expected. This is already suggested by the fact that one has different types of curves near the origin in figure 2. More explicitly, one obtains that

$$K_2\beta^{-1} \approx \tfrac{1}{4}\beta^{-1}\ln(p_+/p_-) + \tfrac{1}{2}(J_+ - J_-) \qquad \text{as } \beta \to \infty. \tag{10}$$



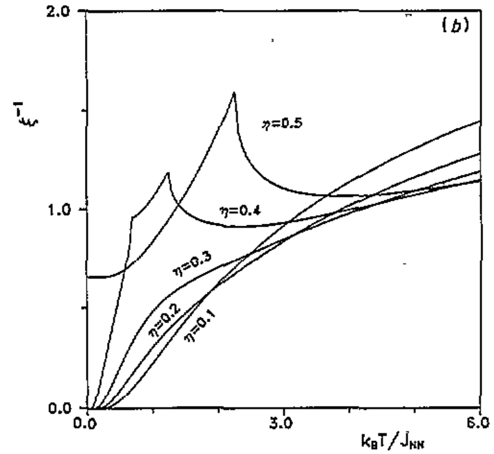


Figure 2. The inverse correlation length as a function of k_BT/J_{NN} for different values of $p_+, p_-, \gamma \equiv J_+/J_-$ and $\eta \equiv -\frac{1}{2}(J_+ - J_-)/J_{NN}$ (as indicated). This corresponds to exact results for the one-dimensional system. (*a*) The case $p_+ = \frac{1}{4}$, $p_- = \frac{1}{2}$, $\gamma = \frac{3}{4}$; this is qualitatively similar to the ordinary case (which corresponds to $p_+ = p_- = \frac{1}{2}$ for any γ). The minima in ξ locate the *disorder line* where the decay of correlations changes from monotonous to oscillating. As compared to the ordinary case, these minima occur here at lower temperature for a given value of η. The *frustration point* corresponds to $\eta = \frac{1}{2}$, where no thermal critical point occurs (cf. the main text, however). (*b*) The same for $p_+ = \frac{1}{2}$ and $p_- = \frac{1}{4}$, revealing the important qualitative differences induced by such variation in the values of the parameters p_+ and p_-; *note that the horizontal axis has a different scale than for that in part* (*a*). (*c*) The same for $p_+ = p_- = 0.10$ and $\gamma = \frac{1}{4}$.

Thus, one may say that the correlation length diverges either as $\xi \sim \frac{1}{2}(p_+/p_-)\tau^{-1}$ or as $\xi \sim 2(p_-/p_+)\tau^{-1/2}$ for η larger or smaller than $\frac{1}{2}$, respectively; here, $\tau \equiv \exp[-2\beta|J_+ - J_- + J_{NN}|]$. That is, one may (only) find a correspondence between these cases and the canonical cases $\nu = 1$ and $\nu = \frac{1}{2}$ by introducing a *distance* τ to the critical point that is system-dependent. The situation is also interesting for $\eta = \frac{1}{2}$ (the frustration point). Then, the usual *thermal* critical point does not occur, but one gets

$$\xi^{-1} \sim \ln\{[1 + (1 + 4p_-/p_+)^{1/2}]/[1 + (1 - 4p_-/p_+)^{1/2}]\} \tag{11a}$$

$$\xi^{-1} \sim \ln\{(p_+/4p_-)^{1/2} + [1 + p_+/4p_-]^{1/2}\} \tag{11b}$$

for $p_+/p_- > 4$ and $p_+/p_- < 4$, respectively. This reflects a sort of *percolative* critical behaviour for $\beta \to \infty$ *and* either $p_- \to 0$ or $p_+ \to 0$, respectively. In contrast, one has that $\xi^{-1} \sim \ln(1 + \sqrt{2})$ for $p_+ = 4p_-$. Further thermodynamic quantities may easily be obtained from (6) and (7) by standard procedures.

To obtain reliable information from (6) for $d > 1$ is more difficult. One may try to use known analytical results for the two-dimensional ordinary ANNNI case. The

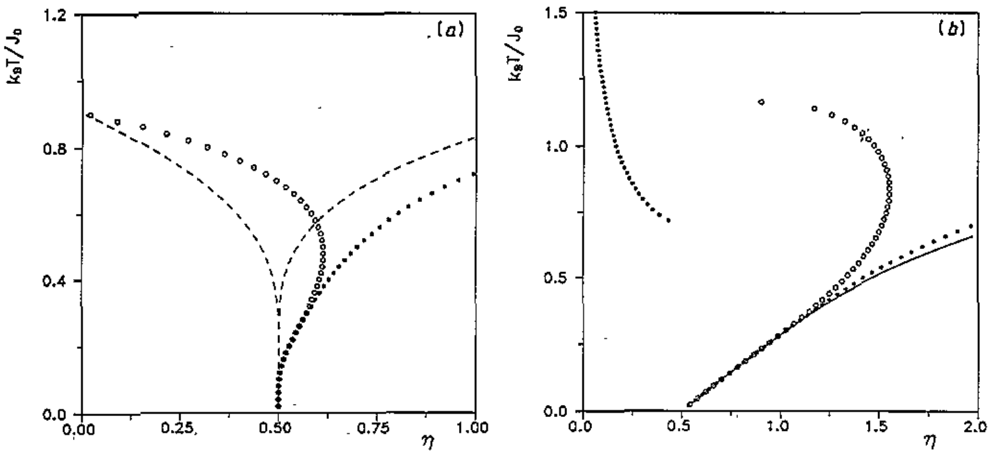


Figure 3. The phase diagram for the two-dimensional system obtained for $J_{NN}=J_0/10$ after using our exact result (6*b*) in (12) (open circles), (13) (full line), and (14) (asterisks). (*a*) The case $\gamma=\frac{3}{4}$ and $p_+=p_-=\frac{1}{4}$; the broken line corresponds to the ordinary ANNNI model. (*b*) The case $\gamma=\frac{3}{4}$, $p_+=\frac{1}{2}$ and $p_-=\frac{1}{4}$; cf. the main text.

combination of both numerical and analytical approximate results shows that the ordinary ANNNI model may be in one of four different phases:

(*a*) a paramagnetic phase occurs at high enough T for any $\alpha \equiv -J_{NNN}/J_{NN}$;
(*b*) a ferromagnetic phase, for $\alpha<\frac{1}{2}$ at low T;
(*c*) the so-called (2,2) antiphase in which two spins up alternate with two spins down in the ground state, for $\alpha>\frac{1}{2}$ at low T (cf. figure 3(*a*) where broken lines represent the boundaries for the above three phases); and
(*d*) an incommensurate modulated phase with continuously varying wavevector for $\frac{1}{2}<\alpha<\frac{3}{2}$ at moderate T between the paramagnetic phase and the (2, 2) antiphase for $d=2$ (this seems to transform into an infinite sequence of commensurate phases for $d=3$).

It has been estimated (Hornreich *et al* 1979, Kroemer and Pesch 1982) that the boundary of the ferromagnetic phase satisfies

$$\sinh(2K_1+4K_2)\sin(2K_0)=1 \tag{12}$$

and that the boundary of the (2, 2) antiphase fulfils

$$\exp(2K_0)=[1-\exp(4K_2)]/\{[1-\exp(-K_1+2K_2)][1-\exp(K_1+2K_2)]\} \tag{13}$$

for $d=2$. A phenomenological method (Villain and Bak 1981) that confirms the above suggests also the existence of the modulated phase (*d*), and the boundary between (*d*) and (*c*) at low enough temperature appears to be characterized by

$$K_1+2K_2=-2\exp(-2K_0). \tag{14}$$

Here, $K_1=\beta J_{NN}$, $K_2=\beta J_{NNN}$ and $K_0=\beta J_0$; for comparison purposes, we consider now transversal NN interactions of strength $J_0\neq J_{NN}$ in our system also. It is questionable whether one may simply substitute (6) in (12)–(14). (That is, the latter refer to (4) when both J_{NN} and J_{NNN} are constant but might not hold for more involved coupling constants such as the ones here.) Assuming this is allowed, we obtain the situation illustrated in figure 3. Figure 3(*a*) depicts departures of the phase diagram from the ordinary case for one of the simplest versions of the generalized model. Figure 3(*b*) reveals that the combination of (6) and (13) seems to induce a novel boundary for any $\gamma > p_+/p_-$ at high temperature. More precisely, the case in figure 3(*b*) exhibits a solution for $\eta < 0.5$, a different one for $\eta > 0.55$, and it seems that two solutions occur for $0.50 < \eta < 0.55$ and $0.70 < 1/K_0 < 0.71$. The new solutions seem to arise due to the fact that K_2 may change sign at high T; it could be just an artefact related to our assumption that (13) applies to the generalized model.

We remark that two different temperatures may be defined in our generalized kinetic model of dimension d although (2) satisfies detailed balance for (5). That is, besides T which represents the canonical temperature associated with the NN term of $E(s)$, one may define $T_{\text{eff}} = K_2/\beta$ associated with the NNN term; the latter has a complex dependence on T, in general. In fact, one cannot construct a canonical steady probability distribution with a unique parameter that has the role of an effective temperature. (For the ordinary one-dimensional system, one may interpret T_{eff} as an effective temperature and J_{NN} as an external field by substituting $s_z s_{z+1} = \sigma_z$; such a simple mapping no longer holds for our kinetic systems, however). Therefore, it could be naive to classify the steady states that follow from (1) to (4) with (5) as ordinary canonical equilibrium states for any d. This is also shown by the one-dimensional case above whose behaviour in terms of T is rather intricate.

The behaviour described above suggests that our model might have an interesting macroscopic, e.g. critical, behaviour for $d = 2$, perhaps even for the quasi-canonical case that corresponds to (5). Unfortunately, one does not have any exact analytical method here to obtain critical behaviour for $d > 1$; in fact, most results dealing appropriately with fluctuations for the ordinary ANNNI model are numerical. Anyhow, it is quite interesting that one may state the result (6) for any d as long as the effective transition rate (2) is a composition of exponential functions as in (5). As one may be convinced, this is related to the fact that the exponential structure of the rate simplifies notably the interactions between a given spin and its surroundings.

As a final remark, we recall two qualitative features of the generalized model that are not shared by the ordinary ANNNI case. The two cusps exhibited sometimes by the correlation length in figures 2(*b*) and 2(*c*) are related to the T dependence of the effective NNN interaction K_2/β in (6*b*). Anyhow, important differences with the ordinary case are expected given that the specific dependence of K_2/β on temperature depends on the values for η and γ, and K_2 may even change sign as T is varied. On the other hand, the significance of the novel upper band in the ferromagnetic phase boundary of figure 3(*b*) is unclear to us at the present time, as discussed above. In fact, a better understanding of the present system for $d > 1$ requires further analysis; we are presently studying this system numerically.

Acknowledgments

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References

Elliot R J 1961 *Phys. Rev.* **124** 346
Garrido P L and Marro J 1989 *Phys. Rev. Lett.* **62** 1929

—— 1994 *J. Stat. Phys.* **71** in press
Glauber R J 1963 *J. Math. Phys.* **4** 294
Hornreich R M, Liebmann R, Schuster H G and Selke W 1979 *Z. Phys.* B **35** 91
Kroemer J and Pesch W 1982 *J. Phys. A: Math. Gen.* **15** L25
Liebmann R 1986 *Statistical Mechanics of Periodic Frustrated Ising Systems* (*Lecture Notes in Physics* **251**) (Berlin: Springer)
López-Lacomba A I, Garrido P L and Marro J 1990 *J. Phys. A: Math. Gen.* **23** 3809
Selke W 1992 *Phase Transitions and Critical Phenomena* vol 15, ed C Domb and J L Lebowitz (London: Academic) pp 2–72
Stephenson J 1970 *Phys. Rev.* B **1** 4405
Tanaka K, Morita T and Hiroike K 1987 *Progr. Theor. Phys.* **77** 68
Villain J and Bak P 1981 *J. Phys.* (*Paris*) **42** 657